

Area I of Computer Engineering (ECE 15A, ECE 152A)

Your score will depend on (1) solidness of your knowledge, (2) depth and insight that you develop to solve the problems.

UNDERSTANDING DUALITY

(a) Find the dual of the following Boolean function:

$$F = A + B'$$

(b) Find the dual of the following Boolean function:

$$G = 1 + AB$$

(c) We have a very clear picture of the complement of a Boolean function on the Boolean cube. We mark each vertex of the Boolean cube with 0 or 1, depending on the value of  $F$  for that row in the truth table of  $F$ . The complement of  $F$  on the Boolean cube will have those values reversed (0  $\rightarrow$  1, 1  $\rightarrow$  0).

When we take the dual of  $F$ , what is geometrically happening on the Boolean cube? (Start with  $F$ 's representation on the Boolean cube, and give a geometric interpretation of how we find the dual. Your explanation should be as simple as possible yet precise. Give your answer for a 3-variable Boolean function  $F$ .)

(d) If an  $n$ -variable Boolean function  $F$  has  $K$  on-terms (i.e. the number of rows of the truth table for which  $F$  is TRUE), then how many on-terms does the dual of  $F$  have?

(e) How many 3-variable self-dual Boolean functions are there? (A self-dual Boolean function  $F$  satisfies  $F = F^D$ ). Write down all these functions. (Hint: Use the geometric insights developed in the previous parts.)