Department of Electrical & Computer Engineering
University of California, Santa Barbara

ECE 240A
Winter 2010
Shynk
H.O. #16

HOMEWORK #5

Due Friday, February 19, 2010 (5:00 p.m.)

Reading: Lessons A, 10, and 11

Problems:

1. Problem 9.5
2. Problems A.1 and A.2
3. Let $z_1, \ldots, z_N$ be i.i.d. random variables distributed according to $N(0, \sigma^2)$. Determine whether or not each statistic below is sufficient for σ^2, and rank them in terms of increasing data reduction. Give your reasoning.

   (i) $T_1 = (z_1, \ldots, z_N)$
   (ii) $T_2 = (z_1^2, \ldots, z_N^2)$
   (iii) $T_3 = (z_1^2 + \cdots + z_m^2, z_{m+1}^2 + \cdots + z_N^2)$
   (iv) $T_4 = z_1^2 + \cdots + z_N^2$

4. Let $z_1, \ldots, z_N$ be i.i.d. random variables drawn according to the uniform distribution $U(\theta_1, \theta_2)$ where $-\infty < \theta_1 < \theta_2 < \infty$.

   (a) Show that $T = (z_{(1)}, z_{(N)})$ (a function of the order statistics) is minimal sufficient.
   (b) Show that T from part (a) is complete.

5. Let $z_1, \ldots, z_N$ be i.i.d. random variables drawn according to the Poisson distribution with parameter λ. Use Method 1 and then Method 2 to find the UMVU estimators of the following:

   (a) λ^k (for any positive integer k)
   (b) $e^{-\lambda}$

6. Suppose that z_1 and z_2 are independently drawn from a Poisson distribution with parameter θ. Define the statistic $T = z_1 + z_2$. Using the definition of a sufficient statistic (not the factorization theorem), show that T is a sufficient statistic for θ.