

HOMEWORK #1

Due Friday, January 15, 2010 (5:00 p.m.)

Reading: Lessons 1, 2, and 3

Problems:

1. Problems 2.5, 2.6, and 2.10
2. Problems 3.6 and 3.7
3. Derive the optimal estimator $h(Y)$ that minimizes the mean *absolute* error given by

$$\text{MAE} = E[|X - h(Y)|].$$

Hint: Use the expression

$$E[|X - a|] = \int_{-\infty}^a (a - u)f_X(u)du + \int_a^{\infty} (u - a)f_X(u)du.$$

4. Suppose that $Y = X + V$ where X and V are *independent* random variables, V is Gaussian with zero mean and unit variance, and X is Bernoulli: ± 1 with equal probability. Show that the optimal (conditional mean) estimator is

$$\hat{X} = \tanh Y.$$

Hint: Note that

$$f_X(x) = \frac{1}{2}\delta(x + 1) + \frac{1}{2}\delta(x - 1)$$

and

$$f_{Y|X}(y|x) = f_V(y - x).$$